Cambridge
IGCSE

## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education


## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/53
Paper 5 (Core)
May/June 2015
1 hour
Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.
Answer all the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24 .

Answer all the questions.

## INVESTIGATION

T-VALUES

A grid of any length and width 10 is numbered $1,2,3, \ldots$.

|  | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| $\Delta 1$ | 47 | $\Delta 2$ | $\Delta 4$ | $\Delta 5$ | $\Delta 6$ | $\Delta 7$ | $\Delta 4$ | 40 | 50 |

The grid has a letter T placed on it, as shown.
The T has a horizontal bar of length 3 and a vertical bar of length 2 .
The T shown is shape 1 because the number in the top left square of the T is 1 .

T-values are found using this method.

| Method | Calculation of T-value for shape 1 |
| :--- | :---: |
| Square the number at the bottom of the T. | $22^{2}=484$ |
| Multiply together the numbers at each end of the horizontal bar. | $1 \times 3=3$ |
| Take the second answer from the first answer to find the T-value. | $484-3=481$ |

The T-value for shape 1 is $\mathrm{T}_{1}=481$.
This investigation is about finding T-values.

1 (a) Complete this table.

| Shape number <br> $n$ | Working | T -value <br> $\mathrm{T}_{n}$ |
| :---: | :---: | :---: |
| 1 | $22^{2}-1 \times 3=484-3$ | $\mathrm{~T}_{1}=481$ |
| 2 | $23^{2}-2 \times 4=529-8$ | $\mathrm{~T}_{2}=521$ |
| 3 |  | $\mathrm{~T}_{3}=$ |
| 4 |  | $\mathrm{~T}_{4}=$ |
| 5 | $\mathrm{~T}_{5}=$ |  |

(b) Explain how you know that $\mathrm{T}_{6}=681$ without using the method in the table.
(c) When a T is placed at the end of a line, it still has a T -value.

The T "wraps round" like this.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 6 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |

Work out $\mathrm{T}_{9}$.
$\mathrm{T}_{9}=$

2 The T is now placed on this 10 by 10 grid.

|  | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

(a) Work out the greatest T -value for a T that fits completely on this grid.
(b) Complete this statement for the numbers in the grid.

In each row the numbers increase by 1 and in each column the numbers increase by $\qquad$ .. .
(c) Complete the squares in this T using expressions in terms of $n$.

(d) Complete this working to show that $\mathrm{T}_{n}=40 n+441$.

The first line of working has been started for you.

$$
\mathrm{T}_{n}=(n+\ldots \ldots \ldots . .)^{2}-n(n+\ldots \ldots \ldots \ldots)
$$

(e) When $\mathrm{T}_{n}=2641$, find the value of $n$.

$$
n=
$$

$\qquad$
(f) Explain why 840 cannot be a T-value.

3 The T is now placed on a new grid that is 11 squares wide.

|  | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 |
| 45 | 46 | 47 | 48 | 40 | 50 | 51 | 52 | 52 | 54 | 55 |

$T_{1}, T_{2}, T_{3}, T_{4}, T_{5}, \ldots \quad$ form a sequence.
(a) Complete this table.

| Shape number <br> $n$ | 1 | 2 | 3 | 4 | 5 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T -value <br> $\mathrm{T}_{n}$ | 573 |  | 661 | 705 |  |  |

(b) (i) Find a formula, in terms of $n$, for $T_{n}$.

$$
\mathrm{T}_{n}=
$$

$\qquad$
(ii) Show that your formula in part b(i) gives the correct result for $\mathrm{T}_{10}$.

4 A different T is placed on a grid that is $w$ squares wide.
The T has a horizontal bar of length 3 and a vertical bar of length 3 .

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |

Complete the squares in this T using expressions in terms of $n$ and $w$.


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